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FIFTH MONTHLY PROGRESS REPORT

Contract NAS 2-1460

This report summarizes the work accomplished under NASA contract NAS 2-1460 during the time period from October 1 to October 31, 1963. The effort was concentrated in the areas of treating the intersection of shock waves and shock waves with weak vortex sheets with real gas effects, developing the laminar boundary layer solution with real gas, and integrating the laminar boundary layer solution with the characteristic solution.

Inviscid Flow Field Routines

During the month of October, the routine for the intersection of shocks of the same family was assembled for the equilibrium real gas case. This routine computes the single shock resulting from coalescense of the two interesecting shocks, the vortex sheet which is caused by the entropy gradient across streamlines passing directly above and below the intersection point, and also the downstream flowfield. In addition, a test is incorporated to determine if a weak shock or expansion fan is reflected from the point of intersection. If either of these situations occurs, the reflected shock or fan is computed along with the rest of the flow field. It is expected that an expansion fan will result in the majority of cases.

The Mathematical Analysis Group is currently programming a routine recently forwarded to them for computing the mutual deflection caused by the intersection of a shock with a weak vortex sheet. The routines for intersection of shocks of the same family are now being programmed for both the real gas and perfect gas cases. Much of the present effort is being directed toward combining the various procedures required in treating shock intersections (i.e.; field point, shock point, shock intersection, vortex sheet, etc.). Routines which were coded during the past month are the intersection of shocks of the opposite family for a real gas and calculation of inlet spillage drag.

The difficulties mentioned in the September progress report, which involve the curve fitting of inviscid flowfield properties, have been resolved to a large extent. The Fortran IV curve fitting routine has been modified to use double precision techniques. Errors observed in using the improved routine have been on the order of 1½% or less. This procedure is extremely important as it is used to obtain inviscid properties for the boundary layer solution and also is used in the shock point routine when the fluid upstream of the shock is not the free stream.

Numerical difficulties have been encountered in obtaining convergence for several of the routines such as the mesh-cutting routine. Problems of this nature, as well as correction of minor programming errors, continue to occupy a large portion of the effort. The blunt body output for the test case supplied by Ames has been determined using the modified Van Dyke solution. Several runs were necessary as points located between the body and shock often turned out to be subsonic. The inviscid solution for the aforementioned test case is being run at the moment and results will be forwarded in the November progress report.

Viscous Flow Routines

If the body is blunted, the viscous flow in the stagnation region is computed from the laminar boundary layer solution. The procedure is to compute the inviscid flow field from the blunt body program, supplied by Ames, up to the sonic line. Next, the local flow properties from this inviscid solution, are taken at the wall and used in the laminar boundary layer solution. The blunt body region is broken up in small increments $\triangle x/R_b$, where x is measured along the surface, and the boundary layer thickness, δ, displacement thickness δ^*, and heat transfer are computed at each station x from the stagnation point x = 0 to the sonic point $x = x_s$. The initial β at the stagnation point is unity, and the β's away from the stagnation point are computed.

From the above solution the boundary layer characteristics are determined at the input line to the characteristic solution. The body, starting at the input line, is corrected for the boundary layer displacement thickness at each body point computed by the characteristic solution. In this manner the displacement thickness due to the boundary layer is included in the inviscid flow field as the solution progresses downstream. This procedure minimizes the machine storage problem.

These routines have been released to Mathematical Analysis and are presently being programmed. The transition and shock boundary layer interaction routines, however, are still under development. Some of the references used in the shock boundary layer interaction study are References 6 - 8.

Two basic methods exist for solving the boundary layer equations. One is to obtain similar solutions by reducing the partial differential equations to ordinary differential equations through a transformation. The transformation is made such that only one independent variable exists. This actually requires special boundary conditions and is only applicable for special boundary layer problems, i. e., stagnation flow and zero pressure gradient flow, Reference 1 and 2. For an arbitrary pressure gradient local similarity may be assumed where the non-similarity terms are neglected, Reference 2 and 3. Then with the proper boundary conditions and initial conditions, the ordinary differential equations may be solved by numerical methods on a digital computer.

The other method is to integrate the boundary layer equations across the boundary layer, again reducing the partial differential equations to ordinary differential equations with integral parameters. Then with a velocity profile law, proper correlations, and boundary conditions, the equations may be solved again by numerical methods on a digital computer. Since only integral parameters appear in the differential equations, the method is particularly attractive to turbulent boundary layers.

For the present program the laminar boundary layer is treated by the former method and the turbulent boundary layer is naturally treated by the latter method. The details of the turbulent boundary layer solution will be given in a future Progress Report.

A detailed discussion of laminar solution is included below. Subsequent Progress Reports will include detail discussions of various other segments of the program.

The boundary layer flow is assumed to be either in thermodynamic equilibrium or frozen state. For completely thermodynamic equilibrium flow the pressure and density define the remaining flow properties. The Mollier diagram, of which a curve fit was provided by Ames, is used to obtain the thermodynamic properties of the gas. For completely frozen flow, however, the gas properties are not uniquely defined by the pressure and density of the gas but depend also on the molecular concentration of the gas.

The following assumptions are made for both frozen and equilibrium flow in the present analysis.

1. All gas species considered behave as perfect gas species.
2. Radiation effects between the body and gas are negligible.
3. The flow is two-dimensional or axially symmetric.

4. **The flow is steady state.**

5. **Prandtl's boundary layer equations are applicable**

The boundary layer equations, compatible with these assumptions, become

continuity of mass:

$$(1) \quad (\rho u r^j)_x + (\rho v r^j)_y = 0$$

Continuity of mass species:

$$(2) \quad \rho u \alpha_x + \rho v \alpha_y = \left[\rho (D_{12}) \alpha_y \right]_y$$

Momentum equation:

$$(3) \quad \rho u u_x + \rho v u_y = P_x + \tau_y$$

Energy equation:

$$(4) \quad \rho u H_x + \rho v H_y = \left[\frac{\mu}{Pr} H_y + \frac{\mu}{Pr} (Pr - 1)(u u_y) \right]_y$$

with the boundary conditions:

At the wall, $y = 0$,

$u = 0$

$H = H_w$ Heat transfer to wall

$H_y = 0$ Zero heat transfer to the wall.

At the edge of the boundary layer, $y = \delta$,

$u = u_e$

$H = H_e = \text{constant}$

Now following the approach of Reference 3, the transformation of the boundary layer equations (1) - (4) are made with the coordinates

$$(5) \quad \xi = \frac{1}{\mu_w^2} \int_0^x \rho_w \mu_w u_e \, r^{2j} \, dx \quad \text{and}$$

$$\eta = \frac{u_e r^j}{\mu_w \sqrt{2\xi}} \int_0^\delta \rho \, dy$$

Introducing these variables in the boundary layer equations and defining

Velocity ratio:

(6) $$f_\eta = u/u_e$$

Total enthalpy ratio:

(7) $$\mathcal{S} = H/H_e$$

Atomic concentration ratio:

(8) $$Z_A = \alpha/\alpha_e$$ and

Pressure gradient parameter:

(9) $$\beta = \frac{2\,\frac{du_e}{dx}}{\rho_w \mu_w u_e^2 r^{2j}\, he/H_e} \int_0^x \rho_w \mu_w u_e r^{2j} dx$$

the partial differential equations reduce to the following, see Reference 3.

Momentum equation:

(10) $$\varphi f_{\eta\eta\eta} + f_{\eta\eta} f + \varphi_\eta f_{\eta\eta} = \beta\, {}^{h}e/H_e \left(f_\eta^{\,2} - \frac{\rho_e}{\rho}\right)$$

Energy equation:

(11) $$\frac{\varphi}{Pr} \mathcal{S}_{\eta\eta} + \frac{\varphi_\eta}{Pr} \mathcal{S}_\eta + f \mathcal{S}_\eta = \frac{\varphi}{Pr}(1 - Pr) \Big[(1 - he/H_e)$$

$$2(f_\eta f_{\eta\eta})_\eta\Big] + \frac{1 - Pr}{Pr} \Big[(1 - he/H_e)\; 2\, f_\eta f_{\eta\eta}\Big] \varphi_\eta$$

Species equation:

(12) $$\frac{\varphi}{Pr} L_e Z_{A_{\eta\eta}} + \varphi_\eta Z_{A_\eta} + f Z_{A_\eta} = 0$$

with the boundary conditions

$$(13)\quad \begin{aligned} f(0) &= f_\eta(0) = 0 \\ \mathcal{S}(0) &= \mathcal{S}_w \text{ with heat transfer} \\ \mathcal{S}_\eta(0) &= 0 \text{ Zero heat transfer} \\ Z_A(0) &= 0 \text{ Catalytic wall} \\ Z_{A\eta}(0) &= 1. \text{ Non-catalytic wall} \qquad \text{and} \\ f_\eta(\infty) &= \mathcal{S}(\infty) = Z_A(\infty) = 1. \end{aligned}$$

The parameter φ is assumed to obey the correlation of Reference 5, or

$$\varphi = \frac{1.0213\ (\mathcal{S}w/t_E)^{.3329} - .0213}{1.0213\ (h/h_E)^{.3329} - .0213} \qquad \text{where}$$

$$h_E = 2.11918684 \times 10^8 \ \text{ft}^2/\text{sec}^2$$

Frozen Flow

For frozen flow in the boundary layer the density ratio ρ/ρ_e is found from the equations, see Reference 4.

$$(14)\quad \frac{\rho}{\rho_e} = \frac{1+\alpha_e}{1+\alpha_e Z_A}\ \frac{T_e}{T} \qquad \text{and}$$

$$\frac{T}{T_e} = \frac{h/h_e - \alpha_e Z_A h_1^{\circ}/h_e}{(3.5 + 1.5\alpha_e Z_A)\ \frac{kT_e}{2m_1 h_e}}$$

where the constants are

$$h_1^{\circ} = 1.6658086 \times 10^8 \ \text{ft}^2/\text{sec}^2$$

$$k/2m_1 = 1715.577\ \frac{\text{ft}^2}{\text{sec}^2\ {}^{\circ}\text{R}}$$

Equations (10) - (12) with the boundary conditions (13) are solved simultaneously. Since equations (10) and (11) are nonlinear and have two point boundary conditions, special methods must be used to solve these equations. The procedure is to guess for all of the initial conditions, equation (13), and integrate equations (10) - (12) by Adams method from $\eta = 0$ to $\eta \cong 6$. Then the boundary conditions of $\eta = 6$ are checked for unity, equation (13). The initial conditions are perturbed, through an optimization program, until all of the boundary conditions are unity. It is evident that good guesses are required to enable all of the boundary conditions to equal unity in a reasonable number of iterations. It has been found, Reference 4, that the guesses

$$\mathcal{S}_{\eta}(0) = .47 \quad \text{Heat Transfer}$$

$$\mathcal{S}_{\eta}(0) = 0 \quad \text{Zero Heat Transfer}$$

$$Z_{A_{\eta}}(0) = .47 \quad \text{Catalytic Wall}$$

$$Z_{A_{\eta}}(0) = 0 \quad \text{Noncatalytic Wall}$$

are reasonable guesses even for $\beta \neq 0$.

The velocity gradient term, $f_{\eta\eta}(0)$, however, is quite sensitive to the pressure gradient parameter β and a better guess is necessary. The starting point for the laminar boundary layer solution is the stagnation point, $\beta = 1$, and for this case Fay and Riddell, Reference 1, have solved the above equations. Using these solutions for various flow conditions, Fay and Riddel obtained correlations for heat transfer to the wall. Using these correlations and Reynolds analogy, the following relation was obtained.

$$(15) \qquad f_{\eta\eta}(0) = (N_u/\sqrt{R_w})\, Pr^{-1/3}\, r^{-1/2}\, \frac{T_w}{T_e} \sqrt{\frac{h_e}{H_e}}$$

where from Fay & Riddell's results

$N_u/\sqrt{R_w}$	Type of Solution
.38	Equilibrium- Catalytic or noncatalytic wall
.40	Frozen - catalytic wall
.18	Frozen - noncatalytic wall

This relation is presently being used for the initial guess on $f_{\eta\eta}(0)$ at the stagnation point. Away from the stagnation point, the previous initial conditions are used.

After a solution to the boundary layer equations has been obtained at a particular station x, the displacement thickness δ^* and heat transfer Nu/(Pr Re) are computed and output, where the displacement thickness is

$$(16) \qquad \delta^* = \frac{\mu_w \sqrt{2\xi}}{\rho_e u_e r^j} \int_0^{\eta_t} \left(1 - \frac{y}{r^j} \cos\theta\right) \left(\frac{\rho_e}{\rho} - f_\eta\right) d\eta \qquad \text{and}$$

$$y = \frac{\mu_w \sqrt{2\xi}}{\rho_e u_e r^j} \int_0^{\eta} \frac{\rho_e}{\rho} \, d\eta$$

and the heat transfer is

$$(17) \qquad \frac{Nu}{Pr\,Re} = \frac{\rho_w u_e H_e r^j}{Pr \sqrt{2\xi}\,\rho_e H_e^*} \left\{ g_\eta(0) + \alpha_e \frac{h_1^o}{H_e} \left[(Le - 1)\, Z_{A\eta}(0) \right]_w \right\}_w$$

where

$$H_e^* = H_e - h_w$$

$$Re = \frac{\rho_e u_e x}{\mu_e}$$

Equilibrium Flow

For thermodynamic equilibrium flow in the boundary layer the procedure for obtaining a solution is similar except for the following. Equations (10) and (11) with the boundary conditions

$$(18) \qquad f(0) = f_\eta(0) = 0$$

$$g(0) = g_w \quad \text{with heat transfer}$$

$$g_\eta(0) = 0 \quad \text{zero heat transfer}$$

$$\text{and} \quad f_\eta(\infty) = g(\infty) = 1.$$

The physical properties of the gas are now obtained from the Mollier diagram. Otherwise the procedure is the same as frozen flow.

A simplified block diagram for the laminar boundary layer program for both frozen and equilibrium flow is given in Figure 1.

It is expected that all programming will be completed in November 1963. This will allow approximately two months for complete program check out.

LAMINAR BOUNDARY LAYER PROGRAM

FIGURE 1

INPUT

$P_e \sim$ Psf Le
$\rho_e \sim$ Slugs/ft^3 Pr
$\Theta \sim 0$
$T_w \sim {}^{\circ}R$
$r \sim$ Ft
$H_e \sim$ ft^2/sec^2 j
$\Delta X \sim$ ft $x \sim$ Ft
$\Delta U_e \sim$ fps

Frozen or Equilibrium Flow
Catalytic or Noncatalytic Wall
Heat Transfer or No Heat Transfer

Guesses $f_{\eta\eta}(o)$ See equation (15)

In program $g_\eta(o) = .47$ yaw
$= 0$ zero yaw
$\mathcal{S}_\eta(o) = .47$ Heat Transfer
or $\mathcal{S}_\eta(o) = 0$ No Heat Transfer

↓

From Mollier Diagram
Find T_e. Compute
α_e, h_e, U_e

Catalytic Wall
$Z_A(o) = 0$
$Z_{A\eta}(o) = .47$

Non Cataylytic Wall
$Z_A(o) = 1.$
$Z_{A\eta}(o) = 0$

↓

Determine $\mu_e + \mu_w$
from viscosity Tables
Compute β

↓

Solve System of
Equations

↓

Compute and Print
out δ^*, δ, and NU/Pr
NU/Re Pr)

Right branch:

Determine $\mu_e + \mu_w$ from
Viscosity Tables
Compute β

↓

Solve System of
Equations

↓

Compute and Print
Out δ^*, δ, NU/Pr
NU/Pr Re)

NOMENCLATURE

C_p - specific heat

D_{12} - bimolecular diffusion coefficient

f - velocity ratio, u/u_e

h_1^o - enthalpy of formation of species

h - static enthalpy

h_e - reference enthalpy

H - total enthalpy

j - exponent, $j = 0$ for two-dimensional body; $j = 1$ for body of revolution.

k - thermal conductivity

Le - Lewis number

Nu - Nusselt number

P_t - total pressure

P - static pressure

Pr - Prandtl number $\frac{\mu c_p}{k}$

R - Reynolds number

r - radius of body at revolution

T - absolute temperature

$u, v,$ - velocity components in x and y direction respectively

x - distance along body surface

y - distance normal to body surface

Z_A - mass fraction ratio α/α_e

α - mass fraction of atoms

β - pressure gradient parameter, equation (9)

g	-	total enthalpy ratio, H/H_e
μ	-	viscosity coefficient
ξ, η	-	similarity variables, equations (5)
η_t	-	η @ the edge of the boundary layer
ρ	-	mass density
τ	-	shear stress
φ	-	density viscosity product ratio, $\rho\mu/\rho_w\mu_w$
θ	-	angle between tangent of body and centerline

Subscripts

E	-	evaluated at reference enthalpy H_e and local pressure
e	-	local value external to boundary layer
w	-	evaluated at wall
x, y, η, ξ	-	derviative with respect to x, y, η, ξ.

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